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### Refresher Material

#### Natural Numbers

1 is the smallest Natural number, 0 is the smallest Whole number, and there is no largest or smallest Integer.

Even numbers are multiples of 2. Any even number can be written as  $2n$ , where  $n$  is an integer. ***0 is an even number.***

Odd numbers are numbers which when divided by 2 leave a remainder of 1. Any odd number can be written as  $2n + 1$ , where  $n$  is an integer.

Factors of a given natural number say  $n$ , is another natural number say  $f$  if  $n$  is completely divisible by  $f$ . Ex factors of 18 are 1, 2, 3, 6, 9, 18 and number of factors is 6.

Highest factor of any natural number is number itself and lowest positive factor is 1. 1 is the factor of every natural number.

The number of factors of any natural number is finite.

## Prime Number

A natural number which has exactly 2 factors is a prime number. Ex number 2 has factors 1 and 2 only. Similarly 3,5,7,11,13,17,19,23,29,31,37,41 etc . **1 is not a prime number.**

A number  $n$  is a prime number if it is not divisible by any prime less than  $[\sqrt{n}]$  where  $[\sqrt{n}]$  is the largest natural number less than or equal to  $\sqrt{n}$ .

Any natural number  $n$  can be written in the form and in one unique way only  $n = p^\alpha \times q^\beta \times r^\gamma \dots$ , where  $p, q, r, \dots$  are different primes and  $\alpha, \beta, \gamma, \dots$  are the powers of the prime number respectively.

### Example

$18 = 2^1 \times 3^2$  . Here 2 and 3 are the prime numbers and 1 and 2 are the respective powers of the primes.

The number of factors of any natural number  $n$ , which can be factored as above is  $= (1+\alpha) \times (1+\beta) \times (1+\gamma) \dots$  etc. Thus number of factors of 18 is  $(1+1) \times (1+2) = 6$ .

The number of odd factors will be given as follows. If the given number does not have any power of 2 then number of factors is same as number of odd factors. But if the number has any term like  $2^\alpha$  in its factorization, as product of prime number powers, where  $\alpha \geq 1$ , then exclude the  $(1+\alpha)$  term and calculate the number of odd factors as  $(1+\beta) \times (1+\gamma) \dots$  Etc. Thus number of odd factors of 18 is  $(1+2) = 3$ , viz... 1, 3 and 9.

The sum of all the factors is given by the expression  $(p^{\alpha-1} - 1) \times (q^{\beta-1} - 1) \times (r^{\gamma-1} - 1) / ((p-1) \times (q-1) \times (r-1)) \dots$

Composite number is a number which has more than 2 factors. For example number 18 has 6 factors viz. 1,2,3,6,9,18.

## Remainder

Any whole number say  $m$  is divided by another natural number say  $n$  then there exists numbers  $q$  and  $r$  such that  $m = nxq + r$ . Where  $q$  is known as the **quotient** and  $r$  is known as the **remainder**. For any  $m, n \in \mathbb{N}$  (the set of natural numbers)  $q$  and  $r \in \mathbb{W}$  (the set of whole numbers). Thus  $0 \leq r$

## Class of integers

As discussed above the remainder obtained when any number is divided by say 5 then remainder is either 0, 1, 2, 3, 4 only. Therefore any number can be written as either as  $5k, 5k+1, 5k+2, 5k+3, 5k+4$ . Thus entire set of numbers has been split into 5 non overlapping sets.

## HCF

HCF of any given set of numbers is a number which completely divides each number in the given set and the number is highest such number possible.

## LCM

LCM of any given set of numbers is the smallest such number which is divisible by each number of the given set.

For any 2 given numbers  **$\text{HCF} \times \text{LCM} = \text{Product of the 2 numbers}$** .

**HCF of fractions = HCF of numerators of all the given fractions / LCM of the denominators of all the fractions.**

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## Divisibility Rules

If the last digit of a number is even then number is divisible by 2

If the sum of the digits of a number is divisible by 3 then 9 then number is divisible by 3 and 9 respectively.

If the last 2 digits of the number are divisible by 4 then number is divisible by 4 and if last 3 digits is divisible by 8 then number is divisible by 8.

If the last digit of the number is 0 or 5 then number is divisible by 5

If the sum of digits of the number is divisible by 3 and the last digit is even then number is divisible by 6.

A given number is divisible by 7 if the number of tens in the original number - twice the units digits is divisible by 7 ex . to check whether 343 is divisible by 7 or not , Thus twice the units digit is  $2 \times 3 = 6$  and number of tens in the number is 34 .Therefore 343 is divisible by 7 if  $34 - 6$  is divisible by 7 i.e. 28 is divisible by 7.

A number is divisible by 11 if the difference of the sum of digits occurring the even numbered places and the sum of digits occurring in the odd number of places is divisible by 11.

## Surds and Indices

### Rules of Indices

$$b^l \times b^m \times b^n \dots = b^{l+m+p\dots}$$

$$(p^m)^n = p^{mn}$$

If bases are same then powers are also same. i.e . if  $a^m = a^p$  and if  $a \neq 1$  or  $0$  then it implies  $m = p$ .

The last digit of the square of any number cannot be 2,3,7 or 8.

Any perfect square is of the form either  $4k$  or  $4k-1$  or  $4k+1$ .

Product of any number of even numbers is even and any number of odd numbers is odd.

The product of any  $n$  consecutive natural numbers is divisible by  $n!$  .

### Useful Algebraic identities

$(a^n + b^n)$  is divisible by  $(a+b)$  for all odd values of  $n$ .

$(a^n - b^n)$  is divisible by both  $(a+b)$  and  $(a-b)$  for even values of  $n$ .

$(a^n - b^n)$  is divisible by  $(a-b)$  for all values of  $n$  (both odd and even).

Sum of first  $n$  natural numbers =  $n(n+1)/2$ .

Sum of squares of first  $n$  natural numbers =  $n(n+1)(2n+1)/6$

Sum of the cubes of first  $n$  natural numbers is given by  $\{n(n+1)/2\}^2$  .

### Units digit of any power of a number (Cyclicity)

If we consider the units digit of the powers of 2 i.e.  $2^x$  for different values of  $x$  then we find the unit's digit is 2,4,6,8,2,4,6,8,2,4,6,8....for  $x= 1,2,3,4,5,6,7,8,9,10,11,12,....$ . Similar patterns exist for the unit's digit of other numbers. The results are summarized in the table below.

Number "a"	Unit's digit of $a^x$ , k is an natural number			
	$x=4k+1$	$x=4k+2$	$x=4k+3$	$x=4k$
1	1	1	1	1
2	2	4	6	8
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

## Solved Examples

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### QUESTION

1) The Rightmost non zero digit of  $5670^{5670}$  is

- (a) 7      (b) 1      (c) 3      (d) 9

### SOLUTION

Here we use rule of cyclicity to solve this problem. The first non zero digit to the left of 0 is 7 hence the rightmost non zero digit will be same as the units digit of  $7^{5670}$ . Now next we need to find out the form of 5670. Dividing 5670 by 4 we get 2 as remainder. Hence 5670 is of the form  $4k + 2$ . Therefore the unit's digit is 9. Therefore answer is d

2) The numbers 13409 and 16760 when divided by a 4 digit integer  $n$  leave the same remainder then the value of  $n$  is

- (a) 1127      (b) 1117      (c) 1357      (d) 1547

### SOLUTION

Here 13409 and 16760 on division by  $n$  leave the same remainder hence can be written as  $nxm + r$  and can be written  $nxp + r$ . Therefore subtracting the 2 equations we get  $nx(m-n) = 3351 = 1117 \times 3$  But  $n$  is 4 digit number hence is  $n$  is 1117. Hence answer is b.



